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Part I FORMULAS

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Part I

FORMULAS

THE GREEK ALPHABET

Greek name	Greek letter	
	Lower case	Capital
Alpha	α	A
Beta	β	B
Gamma	γ	Γ
Delta	δ	Δ
Epsilon	ϵ	E
Zeta	ζ	Z
Eta	η	H
Theta	θ	Θ
Iota	ι	I
Kappa	κ	K
Lambda	λ	Λ
Mu	μ	M

Greek name	Greek letter	
	Lower case	Capital
Nu	ν	N
Xi	ξ	Ξ
Omicron	o	O
Pi	π	Π
Rho	ρ	P
Sigma	σ	Σ
Tau	τ	T
Upsilon	υ	Υ
Phi	ϕ	Φ
Chi	χ	X
Psi	ψ	Ψ
Omega	ω	Ω

1 SPECIAL CONSTANTS

1.1 $\pi = 3.14159\ 26535\ 89793\ 23846\ 2643\ldots$

1.2 $e = 2.71828\ 18284\ 59045\ 23536\ 0287\ldots = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$
$=$ natural base of logarithms

1.3 $\sqrt{2} = 1.41421\ 35623\ 73095\ 0488?\ldots$

1.4 $\sqrt{3} = 1.73205\ 08075\ 68877\ 2935\ldots$

1.5 $\sqrt{5} = 2.23606\ 79774\ 99789\ 6964\ldots$

1.6 $\sqrt[3]{2} = 1.25992\ 1050\ldots$

1.7 $\sqrt[3]{3} = 1.44224\ 9570\ldots$

1.8 $\sqrt[5]{2} = 1.14869\ 8355\ldots$

1.9 $\sqrt[5]{3} = 1.24573\ 0940\ldots$

1.10 $e^{\pi} = 23.14069\ 26327\ 79269\ 006\ldots$

1.11 $\pi^{e} = 22.45915\ 77183\ 61045\ 47342\ 715\ldots$

1.12 $e^{e} = 15.15426\ 22414\ 79264\ 190\ldots$

1.13 $\log_{10} 2 = 0.30102\ 99956\ 63981\ 19521\ 37389\ldots$

1.14 $\log_{10} 3 = 0.47712\ 12547\ 19662\ 43729\ 50279\ldots$

1.15 $\log_{10} e = 0.43429\ 44819\ 03251\ 82765\ldots$

1.16 $\log_{10} \pi = 0.49714\ 98726\ 94133\ 85435\ 12683\ldots$

1.17 $\log_{e} 10 = \ln 10 = 2.30258\ 50929\ 94045\ 68401\ 7991\ldots$

1.18 $\log_{e} 2 = \ln 2 = 0.69314\ 71805\ 59945\ 30941\ 7232\ldots$

1.19 $\log_{e} 3 = \ln 3 = 1.09861\ 22886\ 68109\ 69139\ 5245\ldots$

1.20 $\gamma = 0.57721\ 56649\ 01532\ 86060\ 6512\ldots =$ *Euler's constant*
$= \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\ln n\right)$

1.21 $e^{\gamma} = 1.78107\ 24179\ 90197\ 9852\ldots$ [see 1.20]

1.22 $\sqrt{e} = 1.64872\ 12707\ 00128\ 1468\ldots$

1.23 $\sqrt{\pi} = \Gamma(\frac{1}{2}) = 1.77245\ 38509\ 05516\ 02729\ 8167\ldots$
where Γ is the *gamma function* [see pages 101-102].

1.24 $\Gamma(\frac{1}{3}) = 2.67893\ 85347\ 07748\ldots$

1.25 $\Gamma(\frac{1}{4}) = 3.62560\ 99082\ 21908\ldots$

1.26 1 radian $= 180°/\pi = 57.29577\ 95130\ 8232\ldots°$

1.27 $1° = \pi/180$ radians $= 0.01745\ 32925\ 19943\ 29576\ 92\ldots$ radians

2 SPECIAL PRODUCTS and FACTORS

2.1 $(x+y)^2 = x^2 + 2xy + y^2$

2.2 $(x-y)^2 = x^2 - 2xy + y^2$

2.3 $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

2.4 $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

2.5 $(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

2.6 $(x-y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

2.7 $(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

2.8 $(x-y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

2.9 $(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

2.10 $(x-y)^6 = x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$

The results 2.1 to 2.10 above are special cases of the *binomial formula* [see page 3].

2.11 $x^2 - y^2 = (x-y)(x+y)$

2.12 $x^3 - y^3 = (x-y)(x^2+xy+y^2)$

2.13 $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

2.14 $x^4 - y^4 = (x-y)(x+y)(x^2+y^2)$

2.15 $x^5 - y^5 = (x-y)(x^4+x^3y+x^2y^2+xy^3+y^4)$

2.16 $x^5 + y^5 = (x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)$

2.17 $x^6 - y^6 = (x-y)(x+y)(x^2+xy+y^2)(x^2-xy+y^2)$

2.18 $x^4 + x^2y^2 + y^4 = (x^2+xy+y^2)(x^2-xy+y^2)$

2.19 $x^4 + 4y^4 = (x^2+2xy+2y^2)(x^2-2xy+2y^2)$

Some generalizations of the above are given by the following results where n is a positive integer.

2.20
$$\begin{aligned} x^{2n+1} - y^{2n+1} &= (x-y)(x^{2n} + x^{2n-1}y + x^{2n-2}y^2 + \cdots + y^{2n}) \\ &= (x-y)\left(x^2 - 2xy\cos\frac{2\pi}{2n+1} + y^2\right)\left(x^2 - 2xy\cos\frac{4\pi}{2n+1} + y^2\right) \\ &\quad \cdots\left(x^2 - 2xy\cos\frac{2n\pi}{2n+1} + y^2\right) \end{aligned}$$

2.21
$$\begin{aligned} x^{2n+1} + y^{2n+1} &= (x+y)(x^{2n} - x^{2n-1}y + x^{2n-2}y^2 - \cdots + y^{2n}) \\ &= (x+y)\left(x^2 + 2xy\cos\frac{2\pi}{2n+1} + y^2\right)\left(x^2 + 2xy\cos\frac{4\pi}{2n+1} + y^2\right) \\ &\quad \cdots\left(x^2 + 2xy\cos\frac{2n\pi}{2n+1} + y^2\right) \end{aligned}$$

2.22
$$\begin{aligned} x^{2n} - y^{2n} &= (x-y)(x+y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \cdots) \\ &= (x-y)(x+y)\left(x^2 - 2xy\cos\frac{\pi}{n} + y^2\right)\left(x^2 - 2xy\cos\frac{2\pi}{n} + y^2\right) \\ &\quad \cdots\left(x^2 - 2xy\cos\frac{(n-1)\pi}{n} + y^2\right) \end{aligned}$$

2.23
$$\begin{aligned} x^{2n} + y^{2n} &= \left(x^2 + 2xy\cos\frac{\pi}{2n} + y^2\right)\left(x^2 + 2xy\cos\frac{3\pi}{2n} + y^2\right) \\ &\quad \cdots\left(x^2 + 2xy\cos\frac{(2n-1)\pi}{2n} + y^2\right) \end{aligned}$$

3 The BINOMIAL FORMULA and BINOMIAL COEFFICIENTS

FACTORIAL n

If $n = 1, 2, 3, \ldots$ *factorial* n or n *factorial* is defined as

3.1 $$n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$$

We also define *zero factorial* as

3.2 $$0! = 1$$

BINOMIAL FORMULA FOR POSITIVE INTEGRAL n

If $n = 1, 2, 3, \ldots$ then

3.3 $$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \cdots + y^n$$

This is called the *binomial formula.* It can be extended to other values of n and then is an infinite series [see *Binomial Series*, page 110].

BINOMIAL COEFFICIENTS

The result 3.3 can also be written

3.4 $$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \binom{n}{3}x^{n-3}y^3 + \cdots + \binom{n}{n}y^n$$

where the coefficients, called *binomial coefficients*, are given by

3.5 $$\binom{n}{k} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} = \frac{n!}{k!\,(n-k)!} = \binom{n}{n-k}$$

PROPERTIES OF BINOMIAL COEFFICIENTS

3.6 $$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

This leads to *Pascal's triangle* [see page 236].

3.7 $$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n} = 2^n$$

3.8 $$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots (-1)^n \binom{n}{n} = 0$$

3.9 $$\binom{n}{n} + \binom{n+1}{n} + \binom{n+2}{n} + \cdots + \binom{n+m}{n} = \binom{n+m+1}{n+1}$$

3.10 $$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = 2^{n-1}$$

3.11 $$\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$$

3.12 $$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

3.13 $$\binom{m}{0}\binom{n}{p} + \binom{m}{1}\binom{n}{p-1} + \cdots + \binom{m}{p}\binom{n}{0} = \binom{m+n}{p}$$

3.14 $$(1)\binom{n}{1} + (2)\binom{n}{2} + (3)\binom{n}{3} + \cdots + (n)\binom{n}{n} = n2^{n-1}$$

3.15 $$(1)\binom{n}{1} - (2)\binom{n}{2} + (3)\binom{n}{3} - \cdots (-1)^{n+1}(n)\binom{n}{n} = 0$$

MULTINOMIAL FORMULA

3.16 $$(x_1 + x_2 + \cdots + x_p)^n = \sum \frac{n!}{n_1!\, n_2! \cdots n_p!} x_1^{n_1} x_2^{n_2} \cdots x_p^{n_p}$$

where the sum, denoted by Σ, is taken over all nonnegative integers $n_1, n_2, \ldots, n_p$ for which $n_1 + n_2 + \cdots + n_p = n$.

4 GEOMETRIC FORMULAS

RECTANGLE OF LENGTH b AND WIDTH a

4.1 Area $= ab$

4.2 Perimeter $= 2a + 2b$

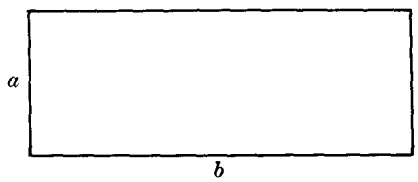

Fig. 4-1

PARALLELOGRAM OF ALTITUDE h AND BASE b

4.3 Area $= bh = ab \sin\theta$

4.4 Perimeter $= 2a + 2b$

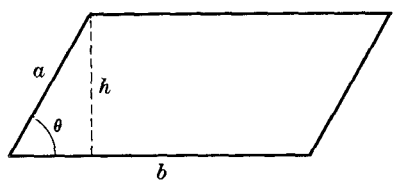

Fig. 4-2

TRIANGLE OF ALTITUDE h AND BASE b

4.5 Area $= \frac{1}{2}bh = \frac{1}{2}ab \sin\theta$

$= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a+b+c) =$ semiperimeter

4.6 Perimeter $= a + b + c$

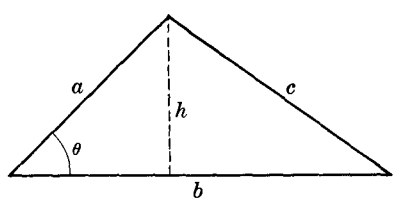

Fig. 4-3

TRAPEZOID OF ALTITUDE h AND PARALLEL SIDES a AND b

4.7 Area $= \frac{1}{2}h(a+b)$

4.8 Perimeter $= a + b + h\left(\frac{1}{\sin\theta} + \frac{1}{\sin\phi}\right)$

$= a + b + h(\csc\theta + \csc\phi)$

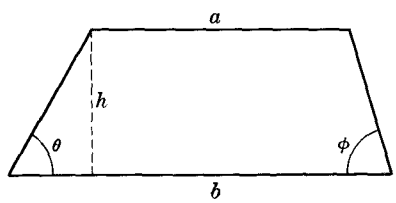

Fig. 4-4

REGULAR POLYGON OF n SIDES EACH OF LENGTH b

4.9 Area $= \frac{1}{4}nb^2 \cot\frac{\pi}{n} = \frac{1}{4}nb^2 \frac{\cos(\pi/n)}{\sin(\pi/n)}$

4.10 Perimeter $= nb$

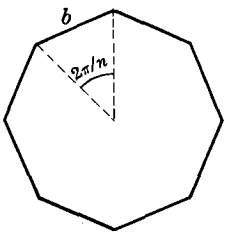

Fig. 4-5

CIRCLE OF RADIUS r

4.11 Area $= \pi r^2$

4.12 Perimeter $= 2\pi r$

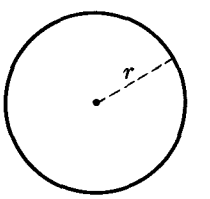

Fig. 4-6

SECTOR OF CIRCLE OF RADIUS r

4.13 Area $= \frac{1}{2}r^2\theta$ [θ in radians]

4.14 Arc length $s = r\theta$

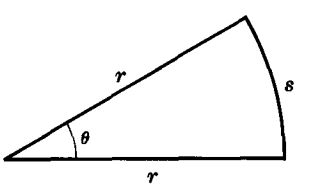

Fig. 4-7

RADIUS OF CIRCLE INSCRIBED IN A TRIANGLE OF SIDES a, b, c

4.15 $$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

where $s = \frac{1}{2}(a+b+c) =$ semiperimeter

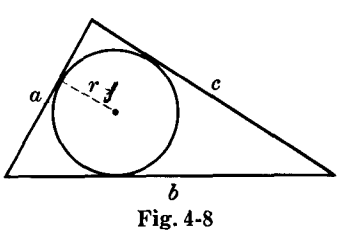

Fig. 4-8

RADIUS OF CIRCLE CIRCUMSCRIBING A TRIANGLE OF SIDES a, b, c

4.16 $$R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

where $s = \frac{1}{2}(a+b+c) =$ semiperimeter

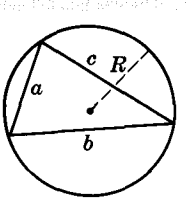

Fig. 4-9

REGULAR POLYGON OF n SIDES INSCRIBED IN CIRCLE OF RADIUS r

4.17 Area $= \frac{1}{2}nr^2 \sin\frac{2\pi}{n} = \frac{1}{2}nr^2 \sin\frac{360^\circ}{n}$

4.18 Perimeter $= 2nr \sin\frac{\pi}{n} = 2nr \sin\frac{180^\circ}{n}$

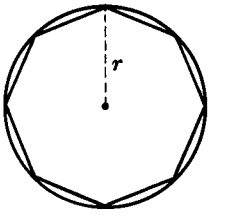

Fig. 4-10

REGULAR POLYGON OF n SIDES CIRCUMSCRIBING A CIRCLE OF RADIUS r

4.19 Area $= nr^2 \tan\frac{\pi}{n} = nr^2 \tan\frac{180^\circ}{n}$

4.20 Perimeter $= 2nr \tan\frac{\pi}{n} = 2nr \tan\frac{180^\circ}{n}$

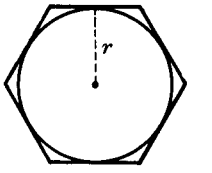

Fig. 4-11

SEGMENT OF CIRCLE OF RADIUS r

4.21 Area of shaded part $= \frac{1}{2}r^2(\theta - \sin\theta)$

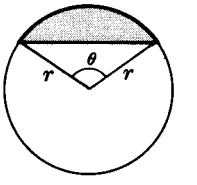

Fig. 4-12

ELLIPSE OF SEMI-MAJOR AXIS a AND SEMI-MINOR AXIS b

4.22 Area $= \pi ab$

4.23 Perimeter $= 4a\int_0^{\pi/2} \sqrt{1 - k^2\sin^2\theta}\, d\theta$

$= 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$ [approximately]

where $k = \sqrt{a^2 - b^2}/a$. See page 254 for numerical tables.

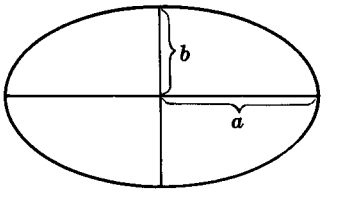

Fig. 4-13

SEGMENT OF A PARABOLA

4.24 Area $= \frac{2}{3}ab$

4.25 Arc length $ABC = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$

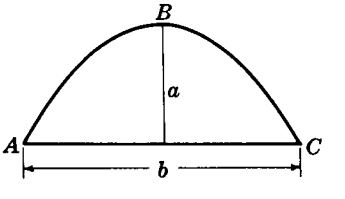

Fig. 4-14

RECTANGULAR PARALLELEPIPED OF LENGTH a, HEIGHT l, WIDTH c

4.26 Volume $= abc$

4.27 Surface area $= 2(ab + ac + bc)$

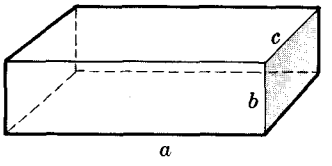

Fig. 4-15

PARALLELEPIPED OF CROSS-SECTIONAL AREA A AND HEIGHT h

4.28 Volume $= Ah = abc \sin\theta$

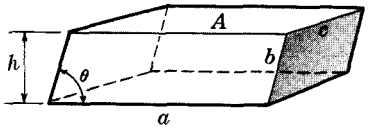

Fig. 4-16

SPHERE OF RADIUS r

4.29 Volume $= \frac{4}{3}\pi r^3$

4.30 Surface area $= 4\pi r^2$

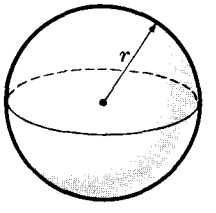

Fig. 4-17

RIGHT CIRCULAR CYLINDER OF RADIUS r AND HEIGHT h

4.31 Volume $= \pi r^2 h$

4.32 Lateral surface area $= 2\pi rh$

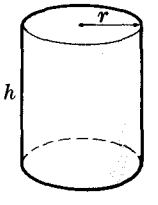

Fig. 4-18

CIRCULAR CYLINDER OF RADIUS r AND SLANT HEIGHT l

4.33 Volume $= \pi r^2 h = \pi r^2 l \sin\theta$

4.34 Lateral surface area $= 2\pi rl = \dfrac{2\pi rh}{\sin\theta} = 2\pi rh \csc\theta$

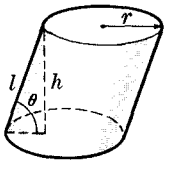

Fig. 4-19

CYLINDER OF CROSS-SECTIONAL AREA A AND SLANT HEIGHT l

4.35 Volume $= Ah = Al \sin\theta$

4.36 Lateral surface area $= pl = \dfrac{ph}{\sin\theta} = ph \csc\theta$

Note that formulas 4.31 to 4.34 are special cases.

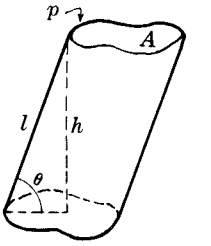

Fig. 4-20

RIGHT CIRCULAR CONE OF RADIUS r AND HEIGHT h

4.37 Volume $= \frac{1}{3}\pi r^2 h$

4.38 Lateral surface area $= \pi r\sqrt{r^2 + h^2} = \pi r l$

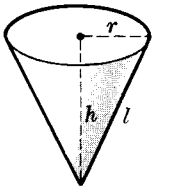

Fig. 4-21

PYRAMID OF BASE AREA A AND HEIGHT h

4.39 Volume $= \frac{1}{3}Ah$

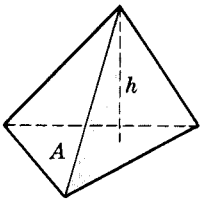

Fig. 4-22

SPHERICAL CAP OF RADIUS r AND HEIGHT h

4.40 Volume (shaded in figure) $= \frac{1}{3}\pi h^2(3r - h)$

4.41 Surface area $= 2\pi r h$

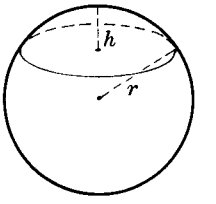

Fig. 4-23

FRUSTRUM OF RIGHT CIRCULAR CONE OF RADII a, b AND HEIGHT h

4.42 Volume $= \frac{1}{3}\pi h(a^2 + ab + b^2)$

4.43 Lateral surface area $= \pi(a + b)\sqrt{h^2 + (b - a)^2}$
$= \pi(a + b)l$

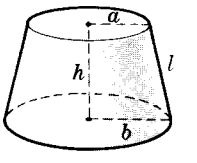

Fig. 4-24

SPHERICAL TRIANGLE OF ANGLES A, B, C ON SPHERE OF RADIUS r

4.44 Area of triangle $ABC = (A + B + C - \pi)r^2$

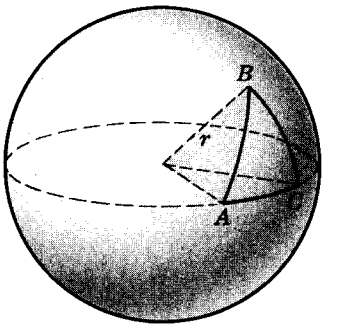

Fig. 4-25

TORUS OF INNER RADIUS a AND OUTER RADIUS b

4.45 Volume $= \frac{1}{4}\pi^2(a + b)(b - a)^2$

4.46 Surface area $= \pi^2(b^2 - a^2)$

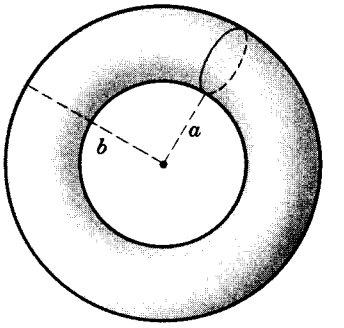

Fig. 4-26

ELLIPSOID OF SEMI-AXES a, b, c

4.47 Volume $= \frac{4}{3}\pi abc$

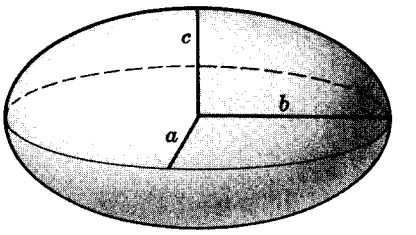

Fig. 4-27

PARABOLOID OF REVOLUTION

4.48 Volume $= \frac{1}{2}\pi b^2 a$

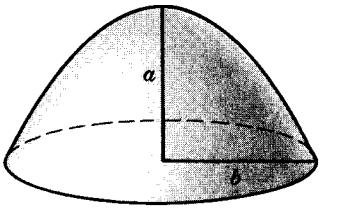

Fig. 4-28